6. P. G. Kroeger and S. Ostrach, "Solution of the two-dimensional problem of freezing with inclusion of convectional effects in the liquid region," Int. J. Heat Mass Transfer, 17, No. 10, 1191-1207 (1974).
7. B. I. Myznikova and E. L. Tarunin, "Natural convection in molten metals during crystallization," in: Mathematical Methods of Analyzing the Processes of Special-Purpose Electrometallurgy [in Russian], Kiev (1976), pp. 129-136.
8. Yu. A. Samoilovich, Ingot Forming [in Russian], Metallurgiya, Moscow (1977).
9. Yu. A. Samoilovich and E. A. Chesnitskaya, "Approximate analytical method of calculating the solidification of ingots," Tr. Vsesoyuz. Nauch.-Issled. Inst. Metallurg. Teplotekh., No. 17, 97-118 (1967).
10. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Nauka, Moscow (1978).
11. G. Z. Gershuni and E. M. Zhukhovitskii, Convective Stability of Incompressible Fluids, Halsted Press (1976).
12. B. I. Medovar, L. M. Stupak, G. A. Boiko, et al., Electroslag Furnaces (B. E. Paton and B. I. Medovar, editors) [in Russian], Naukova Dumka, Kiev (1976).
13. V. V. Panin, O. B. Borovskii, I. S. Ivakhnenko, and S. A. Iodkovskii, "X-ray study of steel smelting by the electroslag process," Izv. Akad. Nauk SSSR, Metall. Gornoe Delo, No. 6, 90-95 (1963).
14. V. V. Panin, O. B. Borovskii, I. S. Ivakhnenko, and S. A. Iodkovskii, "Behavior of droplets and of surface of liquid-metal pool during electroslag smelting," Avtomat. Svarka, No. 2, 72-74 (1964).
15. D. A. Dudko, G. Z. Voloshkevich, I. I. Sumuk-Slyusarenko, and I. I. Lychko, "Study of the electroslag process by means of kinematography and photography through a translucent medium," Avtomat. Svarka, No. 2, 15-17 (1971).
16. Ya. A. Samoilovich and L. N. Yasnitskii, "Algorithm of solution of problems in thermogravitational convection of incompressible viscous fluid by the method of finite elements," Tr. Permsk. Univ. (manuscript No. 1131-80 deposited at All-Union Institute of Scientific and Technical Information March 24, 1980).
17. A. G. Daikovskii, V. I. Polezhaev, and A. I. Fedoseev, "Numerical simulation of transitional and turbulent modes of flow on the basis of transient Navier—Stokes equations," Preprint, Inst. Prikl. Mekh., Akad. Nauk SSSR, No. 101 (1978).

ONE APPROACH TO SOLVING REVERSE PROBLEMS OF HEAT CONDUCTION

Yu. M. Matsevityi

UDC 536.24

Ways are proposed to identify the thermophysical parameters in nonstandard situations (heat transfer in the case low external thermal resistances, thermal conductivity of thin coatings).

The class of reverse problems of heat conduction is rather large, it includes external or boundary problems (determination of boundary conditions from the known mathematical model and available data on the temperature field in the given object), internal or coefficient problems (identification of thermophysical characteristics from available data on the boundary conditions and the temperature field), geometrical problems (determination of the geometrical characteristics of a thermal object), model or inductive problems (refinement of the mathematical model), and finally time or retrospective problems where the initial or simply earlier thermal state is to be determined from available data on the process at later instants of time.

The need to formulate these problems arises from the great difficulties encountered in experimental determination of aforementioned thermophysical parameters and by a natural desire to utilize methods of mathematical simulation for their identification.

Institute of Problems in Machine Design, Academy of Sciences of the Ukrainian SSR, Kharkov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 6, pp. 1119-1123, December, 1981. Original article submitted September 16, 1980.

 0022-0841/81/4106-1384$07.50

In the solution of reverse problems of heat conduction there arise difficulties, some related to the purely technical and methodological aspects of organizing the calculation process, others related to the fundamental nature of the problem. Difficulties of the latter kind arise because reverse problems belong in the category of ill-conditioned ones [1, 2] and, consequently, their solutions are usually not regular (they may be ambiguous, nonconvergent, unstable). Nevertheless, experience in solution of reverse problems with the aid of electrical models [3-5] indicates that in almost all problems solved so far these departures from regularity have been avoided.

This "regularization" of solutions in working with electrical models must not be regarded as characteristic of treatment of reverse problems, inasmuch as this effect is only a consequence of analog devices being less precise than digital computers. The limited precision of electrical models prevents the solution from falling into the window of possible stability loss, the diameter of this window being determined by the maximum measurement error (converted to dimensions of the sought parameters), so that the solution remains stable. In this way, the drawback of analog devices is in a sense turned to an advantage for solution of reverse problems. When the emphasis is not on the means of solving ill-conditioned problems, however, then one must, of course, consider all elements of irregularity of their solutions and especially so on account of the rather high precision of general-purpose digital computers used by most researchers for the solution of such problems. Solution of ill-conditioned problems generally requires the use of regularization procedures or the introduction of constraints under which the solutions will be regular. A special case in this sense is the so-called pseudoreverse problem of heat conduction, where the heat-transfer coefficient and the temperature of the medium or the thermal flux at the surface of a body are sought from known temperatures at points on that surface. It has been demonstrated [2] that such a problem is well-conditioned and its solutions are regular. It thus seems logical to reduce an ill-conditioned reverse problem of heat conduction to a well-conditioned pseudoreverse problem, which will make it unnecessary to test the solutions for uniqueness and stability. We will demonstrate here that this can also be done in the case of an external reverse problem of heat conduction.

Let us consider an infinitely large flat plate (Fig. 1) made of a material with a thermal conductivity λ and known boundary conditions at one of its surfaces (at point A), the heat-transfer coefficient α at its other surface (at point D) to be determined from the temperature inside the plate (at point B a distance δ away from point D) and the temperature T_m of the ambient medium.

We assume that the other boundary surface passes not through point D but through point B. Then determination of the heat-transfer coefficient at that surface (we will call it a fictitious heat-transfer coefficient α_f) will be a pseudoreverse problem, inasmuch as the surface temperature (T_B) is known.

The fictitious external thermal resistance ($1/\alpha_f$) obtained from the solution to this pseudoreverse problem is, naturally, higher than the true thermal resistance $1/\alpha$, since it also includes the internal thermal resistance δ/λ of the discarded layer. Therefore, $1/\alpha = (1/\alpha_f) - \delta/\lambda$, and the true heat-transfer coefficient in the steady-state is

$$\alpha = 1/\left(\frac{1}{\alpha_f} - \frac{\delta}{\lambda}\right). \quad (1)$$

The proposed procedure for reducing an external reverse problem of heat conduction to a pseudoreverse one is applicable not only to one-dimensional problems but also to more complex ones, if the thermal flux is a priori known to be normal to the surface where the boundary conditions are to be identified.

It is often necessary to determine the heat-transfer coefficients in high-intensity processes at a surface. In turbine engineering, for instance, the problem of determining these heat-transfer coefficients α is particularly important in the design of turbines for atomic electric power plants. Components of such turbomachines must operate under intricate conditions of direct contact with wet steam, under conditions where the heat-transfer coefficients increase sharply while the external thermal resistance tends to decrease to zero. In these cases identification of the heat-transfer coefficients through solution of the reverse problem of heat conduction with the aid of analog devices becomes difficult on account of the low precision of the latter (they must handle infinitely large and infinitesimally small quantities here). For this reason, in the case of high-intensity heat transfer

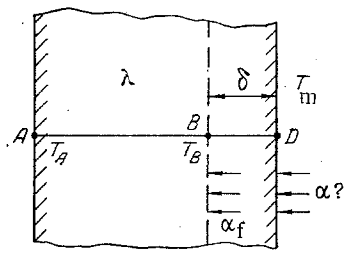

Fig. 1

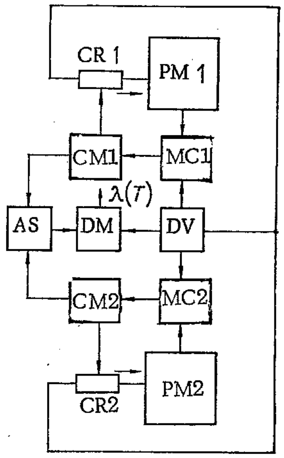

Fig. 2

Fig. 1. Reduction of reverse problem of heat conduction to a pseudoreverse one.

Fig. 2. Schematic block diagram of the device for determining the thermal conductivity of thin coatings.

at a surface, it is expedient to identify the boundary conditions also by way of determining the fictitious heat-transfer coefficient α_f (or the fictitious external thermal resistance $1/\alpha_f$) at some tentative boundary at a distance δ from that surface. The fictitious heat-transfer coefficient obtained through solution of the reverse problem will be smaller than the true heat-transfer coefficient α, which tends to become infinite. One can always match the distance δ so that α_f (and thus also $1/\alpha_f$) will be finite and can be calculated with the aid of analog devices by methods of solution of reverse problems. Most expedient, on the basis of the concepts presented here, would be to shift the boundary so that it will pass through the point where the temperature is known, i.e., so as to make the problem a pseudoreverse one and the solution regular (stable).

Such an approach can also be taken to solution of an internal reverse problem of heat conduction involving identification of the thermal conductivity of objects so small that it is practically impossible to measure their temperature. Such objects are thin films, for instance, widely used for some time now either as principal devices or as coatings on other materials.

The materials of these films (coatings) belong generally in the category of new materials with unknown properties which moreover, owing to the thickness limitation, depend on the thickness as well as on the mode of deposition on the substrate. It therefore is extremely important to know how to determine the thermophysical properties of coatings from readings of the substrate temperature, especially since, as has been just mentioned, measuring the temperature of coatings themselves is often impossible.

We will show how the thermal conductivity of a thin film (coating) can be determined through a parallel solution of two external reverse problems. When the solution of these problems yields the heat-transfer coefficient for a body with coating (α_1) and without coating (α_2), then

$$1/\alpha_1 = 1/\alpha_2 + \delta/\lambda, \tag{2}$$

where δ is the coating thickness.

When the coating thickness is known, then the thermal conductivity can be determined from expression (2) according to the relation

$$\lambda = \delta/\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_2}\right). \tag{3}$$

The researcher is sometimes interested in determining the thickness of a deposited film when the thermophysical properties of the material are known. Then, using the solutions to the same two external reverse problems of heat conduction and expression (3), he can calculate the coating thickness as $\delta = \lambda(1/\alpha_1 - 1/\alpha_2)$.

Electrical simulation [6] can be successfully used for identification of λ (or δ) of a coating and for identification of α in the case of high-intensity heat transfer.

When electric potential and electric current are adopted as the analogs of temperature and thermal flux respectively, then we find that electrical resistance becomes the analog of thermal resistance. The device shown in Fig. 2 facilitates the determination of the thermal conductivity of a coating material from the results of two experiments performed under identical conditions for the same body with and without the coating respectively. For this purpose, the device includes two passive models PM1 and PM2 which are either resistance networks or made of electroconductive paper. Signals at the nodal points of these passive models are fed to the inputs of comparator modules MC1 and MC2, at the second inputs of which there appear voltages coming from voltage divider DV and proportional to the measured (in the two experiments) temperatures at corresponding points of the simulated body.

The mismatch signals are transmitted from the outputs of the comparator modules to the inputs of control modules CM1 and CM2, which adjust the controllable resistors CR1 and CR2 till the mismatch pulses vanish, which will signify that the solution of the reverse problems is completed, i.e., the magnitudes of the adjustable resistances correspond to the thermal resistances $1/\alpha_1$ and $1/\alpha_2$, respectively. The voltages proportional to these thermal resistances appear at the inputs of the adder—subtractor AS, its output signal proportional to the difference $1/\alpha_1 - 1/\alpha_2$ being fed to the input of divider module DM and at the second input of the latter appearing the output voltage of the voltage divider proportional to the coating thickness δ. As a result, a signal is generated at the output of the divider module which corresponds to the thermal conductivity of the coating material λ. By varying the temperature of the ambient medium and measuring the electric potential at the model boundary, which corresponds to the temperature at the body boundary, we can obtain the $\lambda = f(T)$ relation for a given coating material.

The block diagram in Fig. 2 can, if desired, be converted to the block diagram of the algorithm of solution of this problem with the aid of a digital computer.

Thus, the proposed approach to solution of reverse problems of heat conduction, by reducing the problem in many cases to a well-conditioned one, makes it possible to obtain stable solutions without the need for other regularization procedures. It furthermore makes it possible to identify thermophysical parameters when their determination by other methods is either difficult or altogether impossible.

NOTATION

α, heat-transfer coefficient, W/(m^2·°C); λ, thermal conductivity, W/(m·°C); δ, thickness, m; T, temperature, °K; subscript m, ambient medium; and subscript f, a fictitious quantity.

LITERATURE CITED

1. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Ill-Conditioned Problems [in Russian], Nauka, Moscow (1979).
2. O. M. Alifanov, Identification of Heat Transfer Processes in Aircraft [in Russian], Mashinostroenie, Moscow (1979).
3. L. A. Kozdoba, Solutions to Nonlinear Problems of Heat Conduction [in Russian], Naukova Dumka, Kiev (1976).
4. Yu. M. Matsevityi, Electrical Simulation of Nonlinear Problems in Engineering Thermophysics [in Russian], Naukova Dumka, Kiev (1977).
5. Yu. M. Matsevityi, V. E. Prokof'ev, and V. S. Shirokov, Solution of Reverse Problems of Heat Conduction with Electrical Models [in Russian], Naukova Dumka, Kiev (1980).
6. Yu. M. Matsevityi, "Device for simulating the thermal conductivity of thin films," Inventor's Certificate No. 752,383, Byull. Izobret., No. 28 (1980).